Seminar for Discrete Mathematics I -- WiSe17/18

Instructor:
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Schedule:
Tuesday, 14:00
Arnimallee 2 Seminar Room

Weekly Plan

Week 1 (October 17) - Presentation of Topics

During the first week, I will present the topics for the seminar. Since this will take the whole lecture, we will probably meet exceptionally to let everyone present.

Week 2 (October 24) - Species Theory (JPL)

Species Theory is a framework to give a combinatorial approach to generating functions of labeled structures.

Goal:
See the basic definitions and operations.

References: Kerber, Applied finite group actions, Bergeron-Labelle-Leroux

Week 3 (November 7) - Group Actions (JPL)

Once we know how to deal with labeled structures, we can *remove* the labels and define unlabeled structures as equivalence classes under a group action.

Goal:
Revise the orbit-stabilizer and Burnside's Lemma and objects related to group actions

References: Kerber, Applied finite group actions, Bergeron-Labelle-Leroux, Biggs (Ch. 20), Bergeron-Labelle-Leroux (Appendix), Kerber

Week 4 (November 14) - Polya Theory (Evgeniya)

Once we know how to deal with labeled structures, we can *remove* the labels and define unlabeled structures as equivalence classes under a group action.

Goal:

Define weights, the cycle index polynomial, prove Polya's Theorem and give examples

References: Kerber, Applied finite group actions, (Ch. 3.1-2), Bergeron-Labelle-Leroux (Appendix 1), Brualdi (Ch. 14), Beeler (Ch. 8),

Week 5 (November 21) - Tutte polynomial of graphs (Konrad)

The Tutte polynomial of a graph is a two-variable polynomial that generalizes the chromatic polynomial. Further, it is *universal*; any other multiplicative invariant given by deletion/contraction has to be an evaluation of it.

Goal:

Define the polynomial and give some applications

References: arxiv:0803.3079 (https://arxiv.org/pdf/0803.3079)

Week 6 (Nov. 28): Lindström-Gessel-Viennot Lemma (Karolina)

Given a graph G, n starting vertices $S_{1,\ldots,n}$ and n terminal vertices $T_{1,\ldots,n}$, how many n-tuples of vertex-disjoint paths that resp. go from S_i to T_i are there?

Goal:

Give a proof of the Lemma, with examples and Cauchy-Binet Theorem as a consequence

References: Proofs from the book (Ch. 29),

Week 7 (Dec. 5): Kirchhoff's Matrix-Tree Theorem (Dennis)

Cayley's theorem gave the number of spanning trees of the labeled complete graph. What about arbitrary labeled graphs?

Goal:

Give a proof of the Theorem, with examples and Cayley's Theorem as a consequence

References: Harris-Hirst-Mossinghoff (p. 47-) Combinatorics and Graph Theory

Week 8 (Dec. 12): Sperner and Erdös-Ko-Rado Theorems (Claudia)

Given a set $[n]$, what is size of the largest antichain in the poset $(2^{[n]}, \subseteq)$?

A *intersecting family* is a subset of $2^{[n]}$ such that any two member have a nonempty intersection. What is the largest size of an intersecting family of $2^{[n]}$?

Goal:

Give an answer to both questions.

References: Proofs from the book (Ch. 27)

Week 9 (Dec. 19): Kruskal-Katona Theorem (Leo)

Given a simplicial complex with f_i faces of cardinality i, how many faces of cardinality $i-1$ must it have at least?

It is possible to give an answer to this question which will also characterize the f-vector of all simplicial complexes.

Goal:

Give a proof of the Theorem, possibly make a link to faces of convex polytopes

References: Frankl, A new short proof for the Kruskal-Katona theorem. Discrete Math. 48 (1984), no. 2-3, 327–329. Ziegler, Lectures on Polytopes, Section 8.5

Week 10 (Jan. 9): Frankl-Furedi-Kalai Theorem (Iason)

Description:

Given an r-colored simplicial complex with f_i faces of cardinality i,
how many faces of cardinality $i-1$ must it have at least?

Goal:

Give a proof of the Theorem and relate to balanced Cohen-Macaulay complexes.

References: London, Eran, A new proof of the colored Kruskal-Katona theorem. Discrete Math. 126 (1994), no. 1-3, 217–223.

Week 11 (Jan. 16): Boolean Algebras and applications (Niall)

Description:

How to construct a control system using Boolean Algebras? How does a computer *really* add two numbers?

Goal:

Give a formal definition of Boolean algebra (through lattice theory).
Present applications to control systems, computer structure or logic.

References: Gallier, (Ch. 5.13), Grätzer (Ch. 1-2), G.F. South Boolean Algebra and its Uses

Week 12 (Jan. 23): Tamari and Cambrian Lattices (Avail./JPL)

It is possible to give a lattice structure to combinatorial structures
counted by the Catalan numbers. This structure is ubiquitous in
combinatorics and implies deep connections between areas of mathematics.
This Tamari lattice structure can be generalized to the so-called Cambrian lattices.

Goal:

Define the Tamari order (via triangulations and permutations), give some
properties and present its generalization called the Cambrian lattices.

References: Nathan Reading (arxiv:1109.5105)

Week 13 (Jan. 30): Enumeration by Stabilizer Class (Sophia)

Polya's Theorem allows us to count unlabeled structures by weights. Another
modification of Cauchy-Frobenius lemma allows one to count unlabeled structures
with a certain fixed stabilizer subgroup.

Goal:

Present a overview of the proof of Burnside's (real) Theorem and present
small examples.

References: Kerber (Ch. 4.1, and a bit of 4.2)

Week 14 (Feb. 6): Polynomials/Quasi-Polynomials (Avail./JPL)

When studying recurrence relations using the characteristic polynomial
method, if 1 is the only root, the solution is a polynomial. If all roots are
roots of unity, then it is a quasi-polynomial.

Goal:

Give definitions of the notions and give an example for the number of
polygon dissections.

References: Stanley (Ch.4.3-4), P. Lisoněk, Closed forms for the number of polygon dissections, J. Symbolic Comput. 20 (5–6) (1995) 595–601.

Week 15 (Feb. 13): Transfer-Matrix Method (Avail./JPL)

Let $f(n)$ be the number of words $a_1a_2\ldots a_n \in [3]^n$ such that there are no factors of the form 12, 213, 222, 231, or 313. What is the generating function for $f(n)$?

Goal:

Introduce the Transfer-Matrix Method and give an example of how to apply the method.

References: References: Stanley (Ch. 4.7)