Exercise Sheet 9

Discrete Mathematics I - SoSe17

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You should solve all of the exercises below, and select three to four solutions to be submitted and graded. We encourage you to submit in pairs, please remember to

i) indicate the author of **each** individual solution,
ii) the **name of both team members** on the cover sheet,
iii) **read carefully** the question.
iv) **drafts** are not evaluated and worth **0 pt**.

Problem 1

Let G be a graph with n vertices such that $\max_{u\in V} \deg u \leq d$, some some $d \geq 1$. Show that G is an induced subgraph of a d-regular graph H on m vertices where $m \geq n$.

Problem 2

The **Petersen graph** $P_{2,5}$ is the graph with vertex set $V := \binom{[5]}{2}$, i.e. the 2-subsets of $\{1, 2, 3, 4, 5\}$ where two vertices are connected by an edge whenever the corresponding 2-subsets are disjoint. Answer and justify each of the following questions.

a) Is P connected?
b) Is P bipartite?
c) Is P regular?
d) What is the girth of P?
e) What is the diameter of P?
f) What is the length of the largest vertex-simple cycle in P?

Problem 3

What is the maximum number of edges of a graph with $n \geq 1$ vertices and $k \geq 1$ connected components?

Problem 4

Prove that each graph with an even number of vertices has two vertices with an even number of common neighbors.

Problem 5

Prove that a graph of order n with at least $\binom{n-1}{2} + 1$ edges must be connected. Give an example of a disconnected graph of order n with one fewer edge.

Problem 6

How many cycle subgraphs does a connected graph of order n of size n have?

Problem 7

Let G be a graph of $2n$ vertices of girth at least 4. Show that G has at most n^2 edges and give an example of graph realizing this upper bound.

Problem 8

The **intersection graph** of a collection of sets $A_1, A_2, \dots, A_n$ is the graph

$$(V, E) := (\{A_1, \dots, A_n\}, \{\{A_i, A_j\} | A_i \cap A_j \neq \varnothing, \quad i \neq j\}.$$

Draw the intersection graph in the following cases.

a) $A_1 := \{0, 2, 4, 6, 8\}$, $A_2 := \{0, 1, 2, 3, 4\}$, $A_3 := \{1, 3, 5, 7, 9\}$, $A_1 := \{5, 6, 7, 8, 9\}$, $A_1 := \{0, 1, 8, 9\}$.

b) $A_1 := \mathbb{Z} \setminus \mathbb{N}$, $A_2 := \mathbb{Z}$, $A_3 := 2\mathbb{Z}$, $A_4 := 2\mathbb{Z} + 1$, $A_5 := 3\mathbb{Z}$.

c) $A_1 := (-\infty, 0)$, $A_2 := (-1, 0)$, $A_3 := (0, 1)$, $A_4 := (-1, 1)$, $A_5 := (-1, \infty)$, $A_6 := \mathbb{R}$.

Problem 9

Show that in a graph $G = (V, E)$ with $|V| \geq 2$, at least two vertices have the same degree.

Problem 10

Let G be a simple graph and s, t be two distinct vertices of G. Assume that there exists two distinct edge-simple chains from s to t. Show that G contains a vertex-simple cycle.