Exercise Sheet 3

Discrete Mathematics I - SoSe17

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You should try to solve all of the exercises below, and select three to four solutions to be submitted and graded. We encourage you to submit in pairs, please remember to indicate the author of each individual solution.

Problem 1

Show that the number of permutations of $[n]$ is

$$\sum_{(b_1,\dots,b_n)} \frac{n!}{b_1! \cdots b_n! 1^{b_1} \cdots n^{b_n}}$$

where $\sum_{i=1}^{n} i b_i = n$.

Problem 2

Give a bijection between the subsets of $[n-1]$ and the compositions of n.

Problem 3

What is the number of compositions of n into odd parts?

Problem 4

Give a combinatorial proof that the number of set-partitions of $[n]$ such that no two consecutive integers appear in the same block is the Bell number $B(n-1)$.

Problem 5

Let P be a convex polygon with n sides. We add all $\binom{n}{2}$ diagonals and we assume that no three intersect in one point. If we remove all intersection point (and the vertices of P) how many line segments are left?

Problem 6

In how many way can we separate 23 people in three groups of 2, 1 group of 3, 1 group of 4, and two groups of 5?

Problem 7

In how many ways can we distribute

a) 5 unlabeled objects in 3 unlabeled boxes?

b) 5 unlabeled objects in 3 labeled boxes?

c) 5 labeled objects in 3 unlabeled boxes?

d) 5 labeled objects in 3 labeled boxes?

Problem 8

What are the number of integer solutions to the following (in)equalities?

a) $x_1 + x_2 + x_3 + x_4 = 17$ and $x_i \geq 0$,

b) $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ and $x_i \geq 1$,

c) $x_1 + x_2 + x_3 + x_4 = 20$ and $0 < x_1 < 5, x_2 > 5, 0 < x_3 < 6, x_4 > 0$,

d) $x_1 + x_2 + x_3 + x_4 \leq 20$ and $x_i \geq 0$,

e) $x_1 + x_2 + x_3 + x_4 \leq 20$ and $-2 \leq x_1 \leq 8, -2 \leq x_2 \leq 8,\ x_3, x_4 \geq 1$,

f) $ax_1 + x_2 + x_3 = an$ and $a, n \in \mathbb{N}, x_i \geq 0$.

Problem 9

An investor has 20 000 Euros to invest in four different funds. Knowing that he will invest an integer number of kEuros (1000Euros) in each fund, what is the number of strategies if

a) the total 20 000 Euros is invested?

b) the total 20 000 Euros is invested in at least 2 funds?

c) only a part of the money could be invested?

Problem 10

Give a bijection between the set of integer partitions of n into at most k parts and the set of integer partitions of n whose parts do not exceed k.