Exercise Sheet 13

Discrete Mathematics I - SoSe17

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Due date NOT TO BE SUBMITTED

Problem 1

Find orderings of the vertices of the graph of the cube for which the greedy algorithm requires 2, 3, and 4 colors respectively.

Problem 2 (Brooks' Theorem)

Prove that the chromatic number of a connected graph G with maximum vertex-degree Δ is at most Δ, excepted if G is a complete graph or an odd cycle, in which case $\chi(G) = \Delta + 1$.

Problem 3

Prove that the Petersen graph is not planar.

Problem 4

Let G be a 3-connected plane graph such that all its regions are bounded by pentagons or hexagons (they have either 5 or 6 bounding edge-curves)

    a) Use Euler's formula to show that G must have at least 12 pentagonal regions.

    b) Prove that if all vertices have degree 3, then G has exactly 12 pentagonal faces.

Problem 5

Let G be a plane graph with strictly less than 12 regions such that each vertex has degree at least 3.

    a) Use Euler's formula to prove that G has a region bounded by at most four edges.

    b) Give an example to show that part 1) fails if G has 12 regions.