Exercise Sheet 12

Discrete Mathematics I - SoSe17

Lecturer Jean-Philippe Labbé

Tutors Johanna Steinmeyer and Patrick Morris

Due date 12 July 2017 -- 16:00

You should solve all of the exercises below, and select three to four solutions to be submitted and graded. We encourage you to submit in pairs, please remember to

i) indicate the author of **each** individual solution,
ii) the **name of both team members** on the cover sheet,
iii) **read carefully** the question.
iv) **drafts** are not evaluated and worth **0 pt**.

Problem 1

Let G be a connected graph, and let H be obtained from G by adding edges xy iff $dist_G(x, y) = 2$. Prove that H is 2-connected.

Problem 2

Assume that G is a graph with the following property: for every edge xy in G, there are two simple cycles C_1 and C_2 such that their intersection is xy. Prove that G is 3-edge-connected.

Problem 3

Prove that the Petersen graph is 3-edge-connected.

Problem 4

Prove that if $G = (X \cup Y, E)$ is a bipartite graph in which the degree of each vertex in X is not less than the degree of each vertex in Y, then G has a *complete* matching.

Problem 5

A building contractor advertises for a bricklayer, a carpenter, a plumber, and a toolmaker, and receives five applicants: one for the job of bricklayer, one for carpenter, one for bricklayer and plumber and two for plumber and toolmaker.

a) Draw the corresponding bipartite graph.

b) Check whether Hall's condition hold for this bipartite graph.

c) Can all of the jobs be filled by qualified people?

Problem 6

Prove that the size of a maximum matching M in a bipartite graph $G = (X \cup Y, E)$ is $|M| = |X| - d$, where d is the *deficiency* of G, i.e., $d := \max_{A \subseteq X}\{|A| - |J(A)|\}$. (Recall: $J(A) := \{y \in Y : xy \in E, x \in X\}$)

Problem 7

Let $G = (X \cup Y, E)$ be the bipartite graph with $X = \{a, b, c, d, e\}$ and $Y = \{v, w, x, y, z\}$, and $E = \{av, ax, bv, bz, cw, cy, cz, dy, dz, ez\}$. Use the algorithm presented in class to find a complete matching in G, starting from the matching $M = \{av, bz, cy\}$.

Problem 8

Show that the edge set of a bipartite graph with maximal vertex degree Δ is the union of Δ matchings.

Problem 9

Consider the usual 8x8 chessboard. If two white (or two black) squares are removed, then the board can no longer be covered by 31 dominoes (clear?). Show that if one removes any white and any black square, then the board can indeed be covered by 31 dominoes.

Problem 10

How many complete matching can a tree have (not counting the symmetry of the bipartition of the vertices of the tree, i.e., a bipartition of the vertices is fixed)?