Exercise Sheet 10

Discrete Mathematics I - SoSe17

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Due date 28 June 2017 -- 16:00

You should solve all of the exercises below, and select three to four solutions to be submitted and graded. We encourage you to submit in pairs, please remember to

i) indicate the author of **each** individual solution,
ii) the **name of both team members** on the cover sheet,
iii) **read carefully** the question.
iv) **drafts** are not evaluated and worth **0 pt**.

Problem 1

Let $G = (V, E)$ be a simple graph of order n. Show that if G has k connected components, then the number of edges of G satisfies $n - k \leq |E|$.

Problem 2

Prove that, if two distinct cycles of a graph G each contain an edge e, then G has a cycle that does notes contain e.

Problem 3

Part 1) Which of the following graphs are Hamiltonian? semi-Hamiltonian?

a) the complete graph K_5.
b) the complete bipartite graph $K_{2,3}$.
c) the graph of the octahedron.
d) the Hasse diagram (as a graph) of the Tamari lattice for the triangulations of the hexagon.

Part 2) For which value of n is K_n Hamiltonian? Which complete bipartite graph is Hamiltonian? (Explain why!)

Problem 4

Show that if n is odd, it is not possible for a knight to visit all the squares of an $n \times n$ chessboard exactly once by knight's moves and return to its starting point.

Problem 5

Give an example of graph to show that the condition $\deg(v) \geq n/2$ in Dirac's Theorem cannot be replaced by $\deg(v) \geq (n-1)/2$.

Problem 6

Under which conditions does the complete bipartite graph $K_{n,m}$ have an Eulerian cycles? Under what conditions does it have an Eulerian path? (Prove!)

Problem 7

Prove that every tree with at least 2 vertices has at least two leaves, that is two vertices of degree 1.

Problem 8

A **rooted tree** is a tree with a special vertex called **root**. For every vertex v in a rooted tree, there is a unique chain from the root to the vertex v, the neighbors of v not included in this chain are called the **children** of v. How many rooted trees such that every vertex has either two children or no children are there?

Problem 9

Prove that a connected graph with a bridge does not have a Hamiltonian cycle.

Problem 10

Prove that if a tree has a vertex of degree p, then it has at least p leaves.