Exercise Sheet 1

Discrete Mathematics I - SoSe17

Lecturer Jean-Philippe Labbé

Tutors Johanna Steinmeyer and Patrick Morris

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Problem 1

a. Prove that the composition of two injective functions is injective. Prove that the composition of two surjective functions is surjective.

b. Prove that a function $f : A \to B$ is injective if and only if for all functions $g, h : C \to A$, $f \circ g = f \circ h$ implies $g = h$.

c. Prove that a function $f : A \to B$ is surjective if and only if for all functions $g, h : B \to C$, $g \circ f = h \circ f$ implies $g = h$.

Problem 2

a. Give an example of a function $f : A \to A$ such that $f^2 = f \circ f = f$ and f is not the identity function.

b. Prove that if a function $f : A \to A$ is not the identity function and $f^2 = f$, then f is not invertible.

c. Give an example of an invertible function $f : A \to A$, such that $f^3 = f$, yet $f^2 = f \circ f \neq f$.

d. Give an example of a noninvertible function $f : A \to A$, such that $f^3 = f$, yet $f^2 = f \circ f \neq f$.

Problem 3

Prove that the set of odd natural numbers is infinite.

Problem 4

Which is larger $99^{50} + 100^{50}$ or 101^{50}? Why?

Problem 5

A Berlin pizzeria advertises that they offer over a million possible pizzas. How many different toppings must they offer if their advertisement is true?

Problem 6

A *word* is a concatenation of letters. A *palindrome* is a word that reads the same forward and backwards (for example, *civic* and *radar*). Find the number of n letter words that **are not** palindromes. (Hint: consider two cases depending on the parity of n).

Problem 7

Find the number of 7 letter words that end with an "a" or do not contain the letter "a".

Problem 8

Let $n \geq 3$. A group of n people, including Alice, Bob, and Carl, is to be seated around a table. Bob refuses to sit next to either Alice or Carl. Find the number of configurations that respect Bob's restriction.

Problem 9

1. How many different words can be obtained by permuting the letters of the word humuhumunukunukuapuaa?
2. How many license plates of the form *ABC 123*, i.e. three letters followed by three numbers, are there?
3. In how many ways can we select a black square and a white square on a chessboard in such a way that the two squares are not in the same column or the same row?

Problem 10

Assume that $S \subseteq 2^{[8]}$ is such that each subset in S has cardinality 4 and each element of $[8]$ belongs to exactly 3 subsets in S. How many subsets are there in S? Write down such an S.