Exercise Sheet 0

Discrete Mathematics I - SoSe17

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Due date Not to be submitted

Problem 1

Prove that:

- $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$, for all integers $n \in \{1, 2, 3, \dots\}$.
- $(1 + 2 + 3 + \cdots + n)^2 = \frac{n^2(n+1)^2}{4}$, for all integers $n \in \{1, 2, 3, \dots\}$.
- $\log_2(3)$ cannot be expressed as the ratio of integers.

Problem 2

Let $f : \mathbb{N} \to \mathbb{N}$ be a function satisfying $f(n+m) = f(n) + f(m)$ for all $m, n \geq 0$. Prove that there is a constant c such that $f(n) = cn$.

Problem 3

Into how many parts is the plane divided by n straight lines no two of which are parallel and no three of which intersect in a single point?

Problem 4

A messenger arrives in a city of three million inhabitants (before the internet era, or after its collapse), bearing interesting news which he tells to two other people 10 minutes later. Then 10 minutes later each of these people tells the news to two more people (who haven't heard the news yet) and so on, as long as there are people who don't know the news. How long does it take the whole city to learn the news?

Problem 5

Prove that

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

Problem 6

A room contains several people, of whom 6 know English, 7 know French, and 6 know German. Two know both English and French, 4 know both English and German, 3 know both French and German, and one person knows all three languages. How many people are there in the room? How many know only English?

Problem 7

In any calendar year, how many Friday the 13th can there be? What is the smallest number possible?

Problem 8

A manufacturer makes toy cubes for children. Each faces of the cubes are painted one of two colors, blue or red. Some cubes are all blue, some all red, and some have mixture of both colors. How many different kinds of cubes can the manufacturer make?

Problem 9

How many paths are there from one corner of a cube to the opposite corner, each possible path being along three of twelve edges of the cube?

Problem 10

Find a solution in integers to the equation $325x + 26y = 91$.