# Formative Midterm Exam

Discrete Mathematics I - SoSe17

**Due date** 20 June 2017 -- 16:00

First Name: _________________________
Last Name: _________________________
Matricule #: _________________________

This is a Formative Midterm Exam. Follow the instructions below.

1) You should solve all the exercises below **without** usage of any reference material.
2) In order to practice for the Klausur, you should take at most **90** minutes to complete the exercises.
3) Provide **complete** justifications of solutions. You are allowed to cite results proven in class, unless you are asked to prove it.
4) The solutions to the problems should be written **directly on this document**. If necessary, you can join A4 blank sheets for longer solutions.
5) The solution sheets should be handed in **stapled**.
6) Drafts and sketches of solutions should be written on separate sheets and **not submitted**; only final clean solutions should be submitted.
7) Arguments that should not be evaluated should be crossed-out with an "X" or be strikethrough once.

Any submission that do not follow the above rules **will not be evaluated**.

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## Evaluation

| | | |
|---|---|---|
| Problem 1 | | 10 |
| Problem 2 | | 10 |
| Problem 3 | | 15 |
| Problem 4 | | 20 |
| Problem 5 | | 15 |
| Problem 6 | | 15 |
| Problem 7 | | 15 |
| Total | | 100 |

## Problem 1

Let $n \geq 0$. Give a combinatorial proof of the following equality

$$\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}.$$

[ /10 pts]

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## Problem 2

Let $m \geq 1$ and $a_i \in \mathbb{N} \setminus \{0\}$, for all $i \in [m]$. Show that there exist integer numbers $j, k$ with $0 \leq j < k \leq m$ such that $\sum_{i=j+1}^{k} a_i$ is divisible by $m$.

[ /10 pts]

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## Problem 3

Give the number of integer solutions to the following (in)equalities.

a) $x_1 + x_2 + x_3 \leq 6$, where $x_1, x_2, x_3 \geq 0$.

b) $x_1 + 2x_2 + 5x_3 = 22$, where $x_1, x_2, x_3 \geq 1$.

[ /15 pts]

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## Problem 4

Solve the following recurrence relations.

a) $R_n = R_{n-1} + 2R_{n-2} + (-1)^n$, $(n \geq 2)$, where $R_0 = R_1 = 1$.

[ /20 pts]

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## Problem 5

Let $n \in \mathbb{N}$. Give a formula for the number of strings consisting of 0,1, or 2's of length $n$, such that all symbol appears an odd number of times.

[ /15 pts]

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## Problem 6

Prove that every finite join-semilattice with $\hat{0}$ is a lattice.

[ /15 pts]

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## Problem 7

True or False?

a) $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}, \quad \forall k \in \mathbb{Z}$.

b) Let $f : P \to P$ be an order preserving bijection and $|P| < \infty$. The inverse $f^{-1}$ is order preserving.

c) $101^{50}$ is smaller than $99^{50} + 100^{50}$.

d) $1^3 + 2^3 + \cdots + n^3 = \binom{n}{2}^2$.

e) Let $S := \{\varnothing, a, b, aa, ab, ba, bb, aba, abb, aab, bab, abab\}$ and define $s \lhd t \quad \Leftrightarrow$ $s$ is a subsequence of $t$. The poset $(S, \lhd)$ is a lattice.

[ /(3x5) pts, -2pt per wrong answer, 1pt for no answer, minimum: 0pts]

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| | |
|---|---|
| a) | |
| b) | |
| c) | |
| d) | |
| e) | |